Walk-Off Effects in Fabry–Perot Diplexers

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Abstract—Fabry–Perot (FP) resonators working under oblique incidence can be used in the millimeter and far infrared bands as diplexers or channel dropping filters. The response of two-grid Fabry–Perot resonators under Gaussian beam excitation is evaluated by adding the fields of the successive passes of the beam. The results coincide with those obtained from a plane wave expansion of the incident field. Closed form expressions are obtained for the losses due to diffraction walk-off, geometrical walk-off, and mismatch. Excellent agreement is obtained with experiments in the 70–80-GHz band. For a 1-GHz-bandwidth filter, working at an incidence angle of 15°, and an incident beam waist radius of 40 mm, the transmission loss at resonance does not exceed 1 dB. The reflection loss off-resonance is about 0.1 dB. This type of diplexer is particularly useful when used in conjunction with quasioptical guiding systems.

I. INTRODUCTION

THE OPERATION of conventional diplexers or channel dropping filters is based on the resonance properties of cylindrical or ring-type resonators coupled to waveguides (for a review, see [1]). Because these devices are lossy and difficult to construct in the millimeter-wave range, it is of interest to investigate quasioptical systems that can perform similar operations. Quasioptical filters are inexpensive and have low losses. A further advantage is that, because of their large areas, they can handle large powers. Quasioptical diplexers are particularly suitable for use with quasioptical guiding systems such as those used for feeding millimeter-wave antennas or for transmitting information in cities [2].

The simplest type of quasioptical diplexer is a plane-parallel Fabry–Perot (FP) resonator working under oblique incidence. A band of frequency is transmitted through the filter, the rest of the beam being reflected and collected. It is in principle possible to fabricate quasioptical diplexers with two FP resonators operating under normal incidence and 3-dB couplers. This arrangement, which requires tight tolerances, is not discussed here. An FP resonator incorporating two grids behaves essentially as a single pole resonator. If three or more grids are used, the band edges can be made steeper and the in-band ripples arbitrarily small. We limit ourselves in this paper to two-grid resonators.

The main difficulty experienced with the type of diplexer just described results from the walk-off losses, which originate from the incident beam being finite in size. Indeed, only infinite plane waves are "matched" to plane-parallel resonators and fully transmitted at the resonance frequency. For clarity, two kinds of walk-off losses are distinguished: the diffraction walk-off loss and the geometrical walk-off loss. The first kind is due to the expansion of the beam by diffraction as it bounces back and forth between the two grids. This effect is observed even under normal incidence. A related effect is observed with focused ray pencils [3]. It is to be distinguished from the diffraction loss that originates from the introduction of apertures in the resonator [4]. The resonator is here assumed to be much larger in diameter than the incident beam. The second kind of loss is experienced when the beam is incident on the resonator at some angle θ different from zero. This loss is a consequence of the lateral displacement of the beam bouncing back and forth between the grids. The successive passes of the beam do not coincide spatially. This effect is called "geometrical" because it can be understood, in first approximation, on the basis of simple geometrical optics considerations. These two walk-off losses have been observed at optical wavelengths [5].

A general expression for the transmission of mode-degenerate optical resonators (such as the FP) under Gaussian beam excitation has been obtained by one of the present authors [6]. This expression is used in the present work. It is easy to see that, quite generally, reflection on the two grids of the FP resonator in succession amounts to a translation of the beam by a length $2d$ (where d denotes the mirror spacing) directed along the normal to the grid plane. An incident beam is therefore translated, after a round trip in the resonator, by a length $2d \cos \theta$ along its own axis, and by a length $2d \sin \theta$ laterally. The condition that the round-trip phase shift be a multiple of 2π reduces for narrow-band filters to the well-known resonance condition

$$2d \cos \theta = l\lambda \tag{1}$$

where l denotes an integer. The resonance frequency therefore increases with the incidence angle θ. The geometrical walk-off can be neglected if the lateral beam displacement is much smaller than the beam radius ξ_0 after a number of round trips roughly equal to the cavity finesse F (F is defined as the ratio of the free spectral range $c/2d \cos \theta$ divided by the 3-dB bandwidth of the resonance). The geometrical walk-off is negligible if

$$2d \sin \theta F \ll \xi_0. \tag{2}$$

The purpose of this paper is to give accurate expressions of the loss suffered when this condition, (2), is not satisfied, and compare these theoretical results with experiments.

We evaluate the transmission properties of plane-parallel FP resonators under oblique incidence for Gaussian

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beam excitation by adding the fields of the successive passes of the incident beam. This method is formally equivalent to the modal approach, which, for the case of plane-parallel resonators, amounts to performing plane wave expansions of the incident beam. The form of the result and the range of application, however, are different. The multipass method gives the response in the form of an infinite sum. This sum is, in general, more convenient to evaluate than the integral obtained from the plane wave expansion method. Furthermore, the multipass method is applicable in principle to wedged FP resonators [6].

II. APPROXIMATIONS

Two FP diplexers are shown in Fig. 1(a) and (b). Fig. 1(a) shows the measuring setup for a diplexer used in beam guiding systems. Dual-mode or hybrid-mode feeds radiate beams that are collimated by lenses corrected for spherical aberration [7]. Similar systems are used for collecting the transmitted and reflected beams. Fig. 1(b) shows the filter incorporated in a waveguide system.

In order to avoid the grating-like effects that degrade the filter response, the grid periods should always be less than $\lambda/(1 + \sin\theta)$. This is assumed to be the case. It is further assumed that the grid period is small enough, compared to the grid spacing, that the fine structure of the field generated at one grid may be negligible at the other grid. Under such circumstances, each grid behaves as an impedance across a transmission line representing free space. We further assume that the ohmic losses are negligible. The grid is then fully characterized by its field reflection coefficient for plane waves: $\rho \equiv R^{1/2}e^{i\alpha}$. R is the power reflectivity and α the phase angle. Note that ρ is usually a function of the incidence angle θ. Because the angular divergences of the beams that we are considering are small, the variation of ρ over the cross section of the beam can be neglected. ρ is then understood to be the grid reflectivity for a plane wave at the angle of incidence (θ) of the beam axis.

The variation of ρ with frequency depends on the type of grid considered. For a mesh, the power reflectivity of the grid decreases as the frequency is increased. This has the effect of making the cavity finesse smaller and smaller at successive resonances with $l = 1,2,\cdots$ in (1). For an array of conducting squares, on the other hand, the cavity finesse increases with frequency, as long as losses can be neglected. Grids may incorporate both capacitive and inductive elements and have resonance properties of their own. It should be noted that if the grating condition given above is to be approached, the equivalent circuit must be modified. For a capacitive grid, a small inductance must be added in series with the capacitance and, for a mesh, a small capacitance must be added in parallel with the inductance [8].

The incident beam is assumed to have a spherical wavefront and a Gaussian irradiance pattern of the form $\exp(-r^2/\xi^2)$, where r denotes the distance from the axis and ξ the beam radius. ξ has a minimum value ξ_0 along the axis, called the beam waist radius.

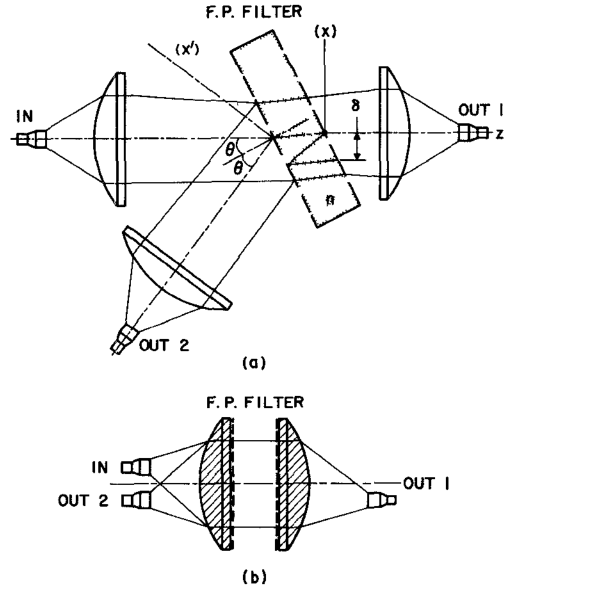

Fig. 1. (a) FP diplexer for beam guiding systems. Measurement system is shown. (b) FP diplexer is incorporated in a waveguide system.

III. GENERAL RESULTS

The results given in this section are applicable to any type of beam (i.e., not necessarily Gaussian) and any lossless optical cavity incorporating two mirrors with uniform reflectivity. Let a wave function ψ be defined by the condition that $\psi\psi^*$ represents the beam irradiance. Because only relative powers are of interest here, ψ can be taken as equal to the electric field, assumed linearly polarized. A more general definition of the wave function is given in [9]. Let ψ_0 denote the incident wave function at some reference plane [plane x in Fig. 1(a)] in the absence of the filter. ψ_0 is normalized to unity in the sense that

$$\iint_{-\infty}^{+\infty} \psi_0^*\psi_0\, dx_1\, dx_2 = 1. \tag{3}$$

If the collecting antenna radiates a wave function ψ_0^* at plane x when used in transmission, the totality of the power is collected in the absence of the filter. We assume that this is the case.

Let now ψ_s denote the wave function of the beam at plane x after s round trips in the resonator, the effect of the grids on the amplitude and phase of the field being omitted. The total field is

$$\psi = t^2 \sum_{s=0}^{\infty} \rho^{2s}\psi_s \tag{4}$$

where t denotes the field transmittivity of the grids, assumed identical, and ρ their field reflectivity. The transmitted power, collected by the receiving antenna, is

$$P_{t0} = \left| \iint_{-\infty}^{+\infty} \psi_0^* \psi \, dx_1 \, dx_2 \right|^2 \equiv | Z |^2 \qquad (5)$$

where vertical bars denote modulus. We have defined

$$Z \equiv T \sum_{s=0}^{\infty} C_s \rho^{2s} \qquad (6)$$

where $T = tt^*$ and C_s is the coupling factor between the incident beam and the beam that has circulated s times in the resonator:

$$C_s \equiv \iint_{-\infty}^{+\infty} \psi_0^* \psi_s \, dx_1 \, dx_2. \qquad (7)$$

Note that, from (3), $C_0 = 1$. Explicit expressions for C_s are given in Section IV for the case where ψ_0 represents a Gaussian beam.

It should be noted that the wave pattern transmitted through the filter usually differs significantly from ψ_0 as a result of the walk-off effects. Part of the power is therefore rejected by the collecting antenna. This mismatch loss can be avoided, at least at one frequency in the band, by reshaping the radiation pattern of the collecting antenna. The total transmitted power is

$$P_t = \iint_{-\infty}^{+\infty} \psi^* \psi \, dx_1 \, dx_2. \qquad (8)$$

ψ being given by an infinite sum (4), the total transmitted power P_t can be expressed as a double sum. Because of the invariance properties of the coupling factor, this double sum can be reduced to a single sum and the formula for the total transmitted power reduces to [6]:

$$P_t = (2 \operatorname{Re} Z - 1 + R)/(1 + R) \qquad (9)$$

where Z is given in (6), and R is the power reflectivity of the grids. To obtain this result we first note that, from (8) and (4),

$$P_t = (1 - R)^2 \sum_{s=0}^{\infty} \sum_{r=0}^{\infty} \rho^{*2s} \rho^{2r} C_{sr} \qquad (10)$$

where we have defined

$$C_{sr} \equiv \iint_{-\infty}^{+\infty} \psi_s^* \psi_r \, dx_1 \, dx_2 = C_{rs}^*. \qquad (11)$$

It can be shown that in a lossless medium the invariance condition

$$C_{sr} = C_{0 \quad r-s} \equiv C_{r-s}, \qquad r \geq s \qquad (12)$$

holds. We have, quite generally,

$$\sum_{s=0}^{\infty} \sum_{r=0}^{\infty} = \sum_{s-r=0}^{\infty} \sum_{r=0}^{\infty} + \sum_{r-s=0}^{\infty} \sum_{s=0}^{\infty} - \sum_{s=r=0}^{\infty}. \qquad (13)$$

Thus (10) can be written

$$P_t = (1 - R)^2 \Big[\sum_{s-r=0}^{\infty} \rho^{*2(s-r)} C_{s-r}^* \sum_{r=0}^{\infty} (\rho\rho^*)^{2r} + \text{c.c.} - \sum_{t=0}^{\infty} (\rho\rho^*)^{2t} \Big] \qquad (14)$$

where c.c. stands for complex conjugate. Introducing Z from its definition (6), the result (9) follows. More generally, if the transformation of the field for a round trip is denoted by $\psi' = dK\psi$, where d is a number and K a unitary operator, it can be shown that the power flowing in the resonator can be expressed as a single sum.

The reflected field at some reference plane [plane x' in Fig. 1(a)] is given by

$$\psi' = \rho\psi_0' + t^2 \sum_{s=1}^{\infty} \rho^{2s-1} \psi_s' \qquad (15)$$

where, as before, ρ and t are the field reflectivity and transmittivity of each grid, ψ_0' is the wave function in the plane x' when the first grid is replaced by a perfect mirror, and ψ_s' is the wave function at x' after s round trips. Assuming that the collecting antenna would radiate a wave function $\psi_0'^*$ at the plane x', the reflected power, collected by this antenna, is

$$P_{r0} = \left| \iint_{-\infty}^{+\infty} \psi_0'^* \psi' \, dx_1' \, dx_2' \right|^2 = | 1 - Z |^2 / R. \qquad (16)$$

In deriving (16), use was made of the assumption that each grid is lossless, i.e., $| \rho |^2 + | t |^2 = 1$, and symmetric, i.e., the phase angles of ρ and t differ by $\pi/2$ [10]. The total reflected power P_r is of course just equal to $1 - P_t$, where P_t is given in (9), since dissipation losses are neglected.

In many important cases, the parameter Z, defined by (6), is real at resonance. In that case, and for high cavity finesses ($T \equiv 1 - R \ll 1$), the loss given by (5) is just twice as large, in decibels, as the loss given in (9). For example, if a 1-dB walk-off loss is suffered in total transmitted power, a 2-dB loss is suffered if the collecting antenna is optimized in the absence of the filter.

It is interesting that the system response is invariant under a translation of the filter. This is a consequence of the observation made before that the beam transformation after s round trips is a translation $2sd$ directed along the normal to the filter plane. This result, in fact, holds true even if the dependence of the grid reflectivity on the incidence angle is taken into account, as one can show on the basis of the plane wave expansion method [11]. This means that it makes no difference whether an FP filter is located at the waist of the incident beam or far from the waist (as long as it is wider than the beam). Calculations can be simplified if we make use of this observation because we can always assume, without loss of generality, that the filter is located at the beam waist. A similar result holds for wedged filters. In that case, the response is invariant under a rotation of the filter about the wedge axis.

IV. GAUSSIAN BEAM EXCITATION

For the case where ψ_0 represents a Gaussian beam, the most convenient way of evaluating the coupling factor C_s introduced in Section III is to represent the incident Gaussian beam by a complex ray. The coupling between two Gaussian beams is then obtained by analogy with the coupling between ray pencils [6].

Let the incident beam axis coincide with the z axis. Upon inspection of Fig. 1(a) and assuming that $n = 1$, we see that after s round trips the beam axis is offset laterally by a length

$$\bar{q}_s = 2sd \sin \theta. \tag{17}$$

The beam axis remains parallel to the z axis because the two grid planes are parallel to one another. As indicated before, we can assume, without loss of generality, that the incident beam waist is located at the (x) plane. The incident Gaussian beam is represented at that plane by a complex ray with position q_0 and slope $\dot{q}_0$:

$$q_0 = \xi_0 \tag{18a}$$

$$\dot{q}_0 = i/k\xi_0 \tag{18b}$$

where ξ_0 denotes the beam waist radius. After s round trips, the complex ray position and slopes are, from the laws of paraxial ray optics,

$$q_s = q_0 + 2sd \cos \theta \dot{q}_0 \tag{19a}$$

$$\dot{q}_s = \dot{q}_0. \tag{19b}$$

The general procedure for obtaining q_s, $\dot{q}_s$ from q_0, and $\dot{q}_0$ is to apply the confluent form of Sylvester's theorem to the 5×5 round-trip ray matrix that characterizes the resonator. In the present case, the result (19) is straightforward.

Because the beam axis remains parallel to its original direction after a round trip, the expression [6, eq. (16)] for the coupling factor C_s simplifies, with the notations of this paper, to

$$C_s = \exp (2iskd \cos \theta) (q_0^*;q_s)^{-1} \times \exp [\tfrac{1}{2}(\bar{q} - \bar{q}_s;q_0^*) (q_0^*;q_s)^{-1}(\bar{q} - \bar{q}_s;q_s)] \tag{20}$$

where we have introduced the complex Lagrange ray invariant

$$(q_1;q_2) \equiv (ik/2) (q_1\dot{q}_2 - \dot{q}_1 q_2). \tag{21}$$

This result (20) can also be obtained by direct integration of the product of the fields of the two Gaussian beams (the incident beam and the beam after s passes in the resonator).

Introducing expressions (17) and (19) into (20), we obtain the coupling factor C_s. Substituting in (6), we get

$$Z = T \sum_{s=0}^{\infty} \rho^{2s} \exp (is\phi) (1 + \tfrac{1}{2}isTD)^{-1} \times \exp [-\tfrac{1}{4}s^2T^2G^2(1 + \tfrac{1}{2}isTD)^{-1}] \tag{22}$$

where

$$\phi \equiv 2kd \cos \theta \tag{23}$$

and where we have defined

$$D \equiv 2d \cos \theta/k\xi_0^2 T \tag{24}$$

$$G \equiv 2d \sin \theta/\xi_0 T. \tag{25}$$

The plane wave expansion method, on the other hand, leads to the following expression for Z:

$$Z = T/\pi \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \exp [-(u^2 + v^2)] \times [1 - \rho^2 \exp (i\phi) \exp \{-iT[uG + (u^2 + v^2)D/2]\}]^{-1}. \tag{26}$$

To obtain this expression we first expand the incident field at plane x in a spectrum of plane waves. These waves have an amplitude $\exp [-\frac{1}{2}(u^2 + v^2)]$, where $u \equiv k_{x_1}\xi_0$ and $v \equiv k_{x_2}\xi_0$, where $k_{x_{1,2}}$ are the components of the wave vector on plane x. The response of an FP to incident plane wave is, as is well known, $t^2[1 - \rho^2 \exp (2ikd \cos \theta')]^{-1}$, where θ' denotes the incidence angle of the plane wave. We have

$$\cos \theta' = \cos \theta(k_z/k) - \sin \theta(k_{x_1}/k)$$

where

$$k_z/k \approx 1 - \tfrac{1}{2}[(k_{x_1}/k)^2 + (k_{x_2}/k)^2]$$

because the angular spread of the beam is small. This response term corresponds to the term in brackets in (26), as one easily verifies. The equivalence of (22) and (26) can be established by applying the binomial expansion to the square bracket expression in (26) and integrating each term.

The transmitted and reflected powers are obtained by summing $C_s\rho^{2s}$ from 0 to ∞, according to (6), and substituting in (5), (9), or (16). These powers depend in general, at some given frequency, on the three parameters D, G, and R. For high cavity finesses, however, that is, when $1 - R \equiv T \ll 1$, only the parameters D and G are significant. When both D and G are small compared with unity, a simple approximate expression for the total transmitted power at resonance is

$$P_t \approx 1 - \tfrac{1}{2}(G^2 + D^2), \qquad D, G \ll 1. \tag{27}$$

The physical meaning of the above relations is more easily understood if we assume that either $G = 0$ or $D = 0$. These approximations are made in Sections V and VI.

V. DIFFRACTION WALK-OFF UNDER NORMAL INCIDENCE

Under normal incidence $\theta = 0$, we have $G = 0$. When $\phi + 2\alpha = 0$, mod 2π, where ϕ is given in (23) and α is the phase angle of ρ, the total transmitted power is, from (22), (6), and (9),

$$P_t(D,R) = T(1+R)^{-1}[2\sum_{s=0}^{\infty} R^s(1+\tfrac{1}{4}s^2T^2D^2)^{-1} - 1]. \quad (28)$$

This sum can be transformed to an integral:

$$P_t(D,R) = \int_0^\infty e^{-w}[1 + 4RT^{-2}\sin^2(\tfrac{1}{4}TDw)]^{-1}\,dw. \quad (29)$$

This second expression (29) can be obtained directly by integrating the transmitted power over the filter area. The variation of P_t with D for various values of T given in (28) or (29) is shown by dashed lines in Fig. 2. Because of the beam divergence, the resonance frequency is slightly higher than the one corresponding to $\phi + 2\alpha = 0$, mod 2π. The actual loss at resonance, obtained from (22), is shown by plain lines in Fig. 2.

For high finesses, P_t is a function of D only. This is easily seen from (29). For high finesses, the sine function in (29) can be replaced by its argument and the T factors then cancel out in the integrand. Also, R can be replaced by unity. We obtain

$$P_t(D) \approx D'[\mathrm{ci}\,(D')\sin(D') - \mathrm{si}\,(D')\cos(D')], \qquad 1 - R \equiv T \ll 1 \quad (30)$$

where si (x) and ci (x) denote the sine and cosine integrals

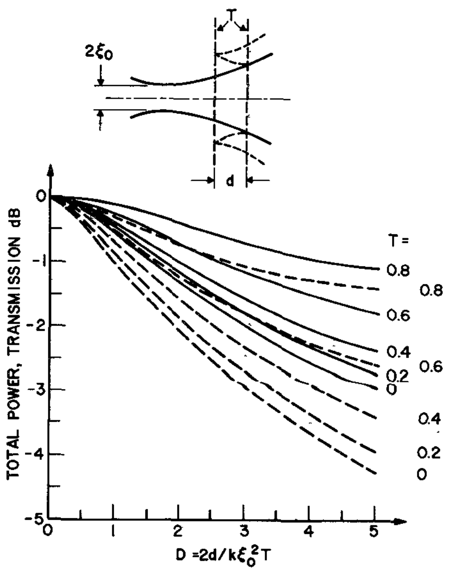

Fig. 2. Diffraction walk-off loss (in decibels) as a function of diffraction parameter D for various grid transmittances (plain lines). ξ_0 denotes the beam waist radius, defined as the $1/e$ point of the irradiance. Dashed lines give the loss at the plane wave resonance frequency ($\phi + 2\alpha = 0$, mod 2π).

and $D' \equiv 2/D$. If we define the quality factor Q of the resonator as the ratio of the center frequency to the 3-dB bandwidth ($Q = kd/T$) we note from Fig. 2 (plain lines) that no more than 1-dB diffraction walk-off loss is suffered when

$$\xi_0/\lambda > 0.2Q^{1/2}, \qquad Q \gg 1. \quad (31)$$

If, for example, a 1-percent bandwidth is desired, the beam waist radius ξ_0 should exceed three wavelengths. This is clearly not a very stringent requirement.

VI. GEOMETRICAL WALK-OFF

Let us now assume that the filter is tilted at some angle θ, but neglect the diffraction walk-off discussed in Section V. Setting now $D = 0$ in (22), we obtain for the filter transmission at resonance ($\phi + 2\alpha = 0$, mod 2π) the expression

$$P_t(G,R) = T(1+R)^{-1}[1 + 2\sum_{s=1}^{\infty} R^s \exp(-\tfrac{1}{4}s^2T^2G^2)]. \quad (32)$$

Because diffraction is neglected, this result coincides with a result given in [6, eq. (28)] for misaligned degenerate cavities. The variation of P_t with G is shown in Fig. (3), with T as a parameter. In the limit of high finesses ($T \ll 1$), P_t depends only on the parameter G and is given by

$$P_t(G) \approx \pi^{1/2}G^{-1}\exp(G^{-2})\,\mathrm{erfc}\,(G^{-1}), \quad 1 - R \equiv T \ll 1 \quad (33)$$

where erfc (x) denotes the complementary error function. We note from Fig. (3) that a 1-dB loss is suffered, as a result of the geometrical walk-off, when

$$\xi_0 = 0.7dF\sin\theta \quad (34)$$

where $F = \pi R^{1/2}T^{-1}$ denotes the cavity finesse. In order that the reflected beam be resolved from the incident beam, the incidence angle θ must be such that

$$\sin\theta \gg 1/k\xi_0. \quad (35)$$

The diplexing operation that we consider is therefore possible only if

$$\xi_0/\lambda \gg 0.2Q^{1/2} \quad (36)$$

where Q denotes, as before, the quality factor of the resonator. This condition is similar to condition (31), which was obtained from different considerations. In most practical cases the incidence angle θ is chosen on the basis of dimensional requirements and is much larger than $(k\xi_0)^{-1}$.

For the case where the collecting antenna is matched to the incident beam in the absence of the filter, we have at resonance, from (22), with $D = 0$, and (5) and (6),

$$P_{t0} = T^2[\sum_{s=0}^{\infty} R^s \exp(-\tfrac{1}{4}s^2T^2G^2)]^2. \quad (37)$$

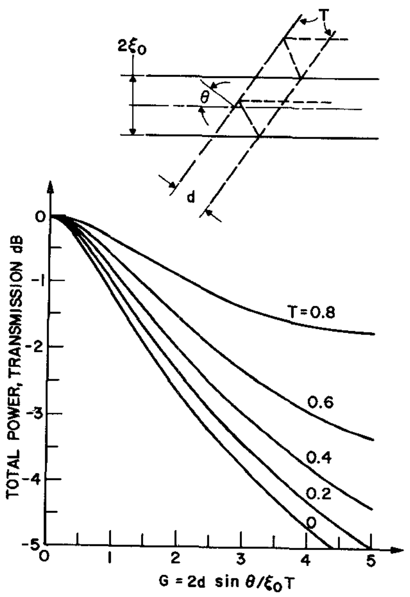

Fig. 3. Geometrical walk-off loss (in decibels) as a function of walk-off parameter G for various grid transmittances. θ denotes the incidence angle.

For small misalignments ($G \ll 1$), (37) is approximately

$$P_{t0} \approx 2.17R(1+R)G^2, \quad \text{dB}. \tag{38}$$

VII. EXPERIMENTAL RESULTS

The predictions for the geometrical walk-off loss made in Section VI have been compared to experimental results obtained in the 70–80-GHz band with an FP filter, 300 mm in diameter, incorporating two planar meshes spaced 8 mm apart. The meshes' dimensions and reflectivities are shown in Fig. 4. The experimental setup is shown in Fig. 1(a), with the electric field perpendicular to the plane of incidence and parallel to one set of mesh wires (TE waves). The beam pattern at the filter location was measured with a scanning device, and a Gaussian beam fitted to the −6-dB points. In all cases, the filter was located at the beam waist. Fig. 5 shows the theoretical response for incidence angles $\theta = 0$, 5, 10, 15, and 20°. The upper curves correspond to the total transmitted power obtained from (32), while the lower curves correspond to the power collected when the antenna is optimized in the absence of the filter [see (37)]. These curves were obtained using (22) with $D = 0$, (6) and (9) for the upper curve, and (5) for the lower curve. Because the diffraction grating effects are negligible, the mesh is represented by a pure inductance L. For the polarization and grid orientation presently considered we have

$$\rho = -(1 - i2\omega L \cos\theta)^{-1}. \tag{39}$$

The values for L and d were obtained by best fitting the theoretical transmission curve to the transmission curve measured under normal incidence ($\theta = 0$). We obtained $\omega L = 0.218$ at resonance (71 GHz), in close agreement

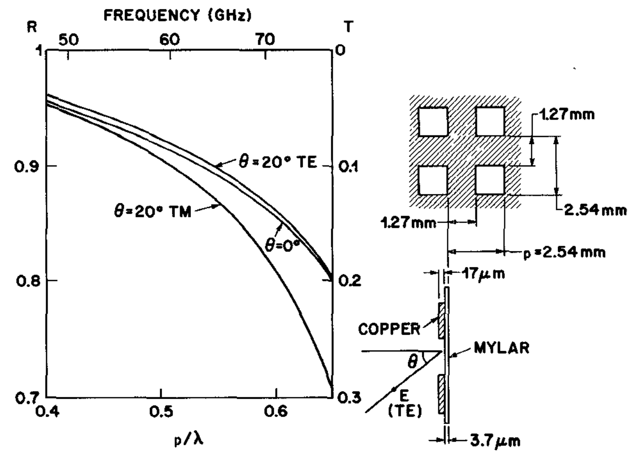

Fig. 4. Curve gives power reflectivity $R \equiv 1 - T$ of mesh used in the experiments as a function of frequency, under normal incidence and under an incidence angle of 20° (TE and TM polarizations). Grid dimensions are also shown.

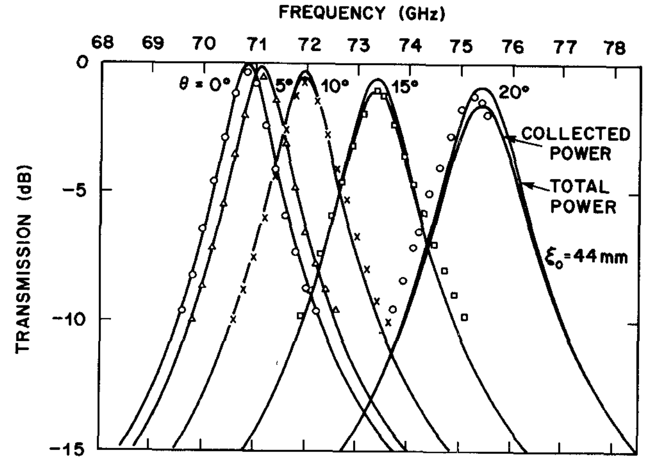

Fig. 5. Plain curves show theoretical response of FP resonator as a function of frequency for various incidence angles. Upper curve gives total transmitted power and lower curve gives power collected when the antenna is optimized in the absence of the filter. Beam waist radius (best fitted to the measured beam) is $\xi_0 = 44$ mm. Filter dimensions are given in text.

with the result given in Fig. 4, and $d = 8.18$ mm. Note that the effective grid spacing d slightly exceeds the mechanical spacing (8 mm). The loss measured with the collecting antenna optimized in the absence of the filter is in excellent agreement with the predicted value, though slightly smaller. This set of measurements was made for a beam waist radius $\xi_0 = 44$ mm (Fig. 5) and $\xi_0 = 24$ mm (Fig. 6). In the latter case the beam was focused to a small spot size by moving the transmitting feed away from the lens. Fig. 5 shows that a 1-GHz 3-dB bandwidth is obtained at $\lambda = 4$ mm. For an incidence angle of 15° (that is, 30° between incident and reflected beams), the walk-off loss does not exceed 1 dB.

The experimental transmitted and reflected powers are shown in Fig. 7 for an incidence angle of 15°. This curve shows that the reflection loss off-resonance is of the order of 0.1 dB. The responses are different for TE and TM

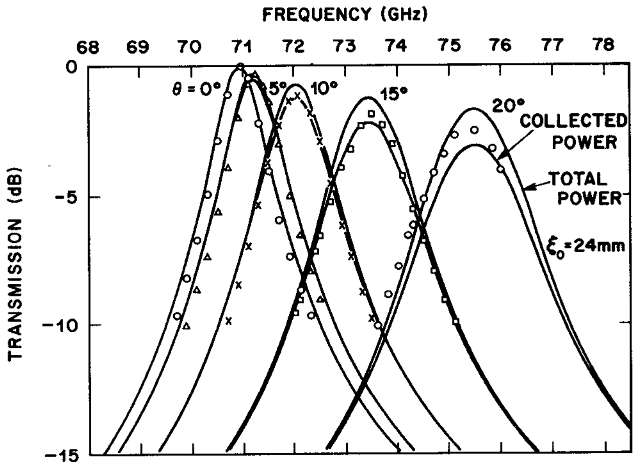

Fig. 6. Continuation of Fig. 5 for a smaller beam waist radius. $\xi_0 = 24$ mm.

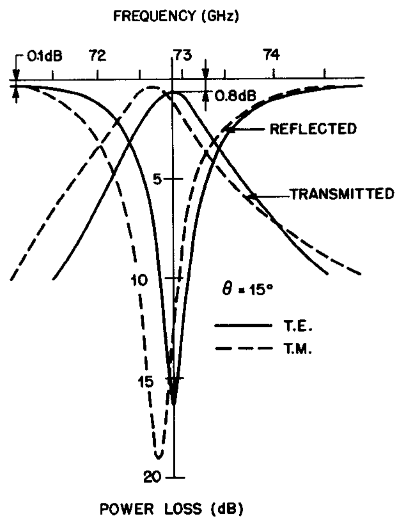

Fig. 7. Measured reflected and transmitted power loss as a function of frequency for an incidence angle of 15° for two polarizations, TE and TM.

polarizations. Therefore incident fields at arbitrary polarizations would be depolarized. We have observed experimentally, however, that if the filter is rotated in its own plane by 45°, the filter response is almost the same for all polarizations.

VIII. FURTHER CONSIDERATIONS

The purpose of this paper was to derive simple formulas for the response of two-grid FP diplexers and verify their validity on typical cases. In order to get a better understanding of the possibilities of quasioptical diplexers, the following qualitative considerations may be useful.

A. Dissipation Losses

The dissipation losses are mainly due to the finite conductivity of the meshes. They have been neglected in this analysis. It should be noted that, for a given bandwidth, the dissipation losses become smaller and smaller as the spacing between the grids is increased. As d is increased, the walk-off losses remain the same because the grid reflectivity (and therefore the effective number of bounces) is reduced. The free spectral range, however, is reduced, and side-resonances appear that may cause difficulties for some applications.

The above statement concerning the effect of the grid spacing on the filter loss rests on the assumption that the grid resistivities are unchanged. In fact, the resistivity of a mesh tends to increase as the reflectivity is reduced because the RF current must flow in conducting tapes of reduced width. This increase in grid resistivity somewhat offsets the benefit of having larger grid spacings. Because capacitive grids do not have this problem, they are to be preferred when the required grid reflectivity is low. Capacitive grids need to be supported by plastic sheets. These sheets, however, can be made thin enough, even for large area filters, not to increase the loss significantly.

B. Wedged FP's

Consider a Gaussian beam incident on a tilted FP resonator far away from the beam waist. Because the beam has a large cross-section area, it is legitimate to evaluate the filter transmission by adding the transmissions of elementary areas, each having a different resonance frequency because of the varying incidence angle. The transmission of the filter is obtained by integration, with the appropriate weighting factor. This approach shows clearly that a better transmission is obtained if the local resonance frequency is made a constant by making the grid spacing nonuniform. Ideally, if one of the grids is plane, the surface of the other grid should be an hyperboloid. If this proves impractical, some improvement can nevertheless be obtained by simply wedging the resonator. In a wedged filter the two grids remain plane, but their planes are not parallel. The general formula given in [6] is applicable to wedged FP resonators. Further numerical analysis will be required, however, in order to evaluate accurately what benefit can be obtained by wedging FP diplexers.

C. Use of Dielectrics in the Filters

The use of high permittivity dielectrics, such as pure alumina ($\epsilon = 9.5$, $\tan\delta \sim 10^{-4}$), in the resonator [as shown in Fig. 1(a)] reduces the walk-off losses for two reasons. First, for a given optical thickness, the grid spacing is reduced by a factor $n = \epsilon^{1/2}$. Secondly, the angle of incidence on the grids inside the filter is reduced because of refraction, and the rate of expansion of the beam is also reduced. The general expressions given previously are easily generalized to the case where the resonator is filled with a homogeneous medium with refractive index n. It should be noted, however, that the grating effect becomes more severe. We must now have $p < \lambda/(n + \sin\theta)$, instead of $p < \lambda/(1 + \sin\theta)$. Furthermore, the fine structure of the field generated at one grid decays more slowly, by a factor n, than in free space. The condition that the grids do not interact through evanescent fields may therefore be difficult to satisfy.

It is possible to alleviate some of these difficulties, while preserving the walk-off reduction, by filling the resonator with uniaxial artificial dielectrics, the optical axis (z') being perpendicular to the grids. If the axial (z') component of the dielectric tensor is large compared with the transverse components, the surface of wave normal is flattened in the z' direction compared with the isotropic case and the group velocities make small angles with the z' axis. This is a favorable feature for reducing the beam rate of expansion, as the theory of beam propagation in anisotropic media shows [9].

D. Gaussian Apertures

For practical reasons, the filter diameter cannot always be made much larger than the incident beam diameter. It is then useful, in order to preserve the beam pattern as much as possible, to introduce Gaussian apertures in the filter. A Gaussian aperture is an absorbing sheet with power transmissivity $\exp[-(r/a)^2]$, where a denotes some effective aperture radius. Vakhimov [12] has shown that mirrors with Gaussian reflectivity are formally equivalent to mirrors with complex curvatures. More generally, a resonator incorporating Gaussian apertures can be described by complex ray matrices. It is therefore not difficult to obtain the response of such filters. The same method is applicable to nonuniform grids provided that the spatial variation of the grid reflectivity be at most quadratic (in decibels) in the x_1, x_2 coordinates.

The above discussion is only suggestive of the arrangements that need be investigated in order to optimize the operation of quasioptical diplexers of the type considered.

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